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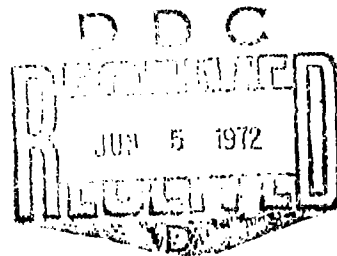

NUMERICAL SOLUTION OF EQUATIONS IN THE TWO PARAMETER
THEORY OF THE BOUNDARY LAYER

BY

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13. ABSTRACT

Numerical solution of problems of the parametric boundary layer theory in a
two-parameter approximation. The partial differential boundary layer equation
is solved by a finite difference method using an implicit difference scheme,
and the results obtained serve as the initial data. The derivatives in the
original equation are replaced by finite difference equations, and the re-
sulting system is solved by a brute force method. A table lists the numerical
results obtained by computer.

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NOTICE

NUMERICAL SOLUTION OF EQUATIONS IN THE TWO-PARAMETER
THEORY OF THE BOUNDARY LAYER

by

Ye. A. Ozerova and L. M. Simuni

1. Formulation of the problem

    In solving problems in the parametric theory of the boundary layer
/1,2/ in the two-parameter approximation, we arrive at the equation

$$\frac{\partial^3 \phi}{\partial \xi^3} + \frac{F + 2f_1}{2B^2}\, \phi\, \frac{\partial^2 \phi}{\partial \xi^2} + \frac{f_1}{B^2}\left[1 - \left(\frac{\partial \phi}{\partial \xi}\right)^2\right] =$$

$$= \frac{Ff_1 + f_2}{B^2}\left(\frac{\partial \phi}{\partial \xi}\,\frac{\partial^2 \phi}{\partial f_1\, \partial \xi} - \frac{\partial \phi}{\partial f_1}\,\frac{\partial^2 \phi}{\partial \xi^2}\right) +$$

$$+ \frac{(f_1 + 2F) f_2}{B^3}\left(\frac{\partial \phi}{\partial \xi}\,\frac{\partial^2 \phi}{\partial f_2\, \partial \xi} - \frac{\partial \phi}{\partial f_2}\,\frac{\partial^2 \phi}{\partial \xi^2}\right),$$

(1)

where

$$F = 2\left\{B\,\frac{\partial^2 \phi}{\partial \xi^2}\bigg|_{\xi=0} - \left[2 + \frac{1}{B}\int_0^\infty\left(1 - \frac{\partial \phi}{\partial \xi}\right)d\xi\right]f_1\right\}$$

with the boundary conditions

$$\phi = \frac{\partial \phi}{\partial \xi} = 0 \text{ for } \xi = 0, \quad \frac{\partial \phi}{\partial \xi} = 1 \text{ as } \xi \to \infty$$

and the auxiliary conditions

$$F \geqslant 0 \; ; \; \frac{\partial^2 \phi}{\partial \xi^2} \geqslant 0.$$

- 1 -

Let us introduce the substitution

$$u = \frac{\partial \phi}{\partial \xi}; \quad v = -\frac{\partial \phi}{\partial f_1}; \quad w = -\frac{\partial \phi}{\partial f_2},$$

then equation (1) reduces to the system

$$
\left.
\begin{aligned}
\frac{\partial^2 u}{\partial \xi^2} + \frac{F + 2f_1}{2B^2}\, \phi\, \frac{\partial u}{\partial \xi} &= \frac{f_1}{B^2}(1 - u^2) - \frac{Ff_1 + f_2}{B^2}\left(u\,\frac{\partial u}{\partial f_1} + \right. \\
&\left. + v\,\frac{\partial u}{\partial \xi} \right) + \frac{(f_1 + 2F)f_2}{B^2}\left(u\,\frac{\partial u}{\partial f_2} + w\,\frac{\partial u}{\partial \xi} \right); \\
\frac{\partial v}{\partial \xi} + \frac{\partial u}{\partial f_1} &= 0; \\
\frac{\partial w}{\partial \xi} + \frac{\partial u}{\partial f_2} &= 0,
\end{aligned}
\right\}
\tag{2}
$$

where

$$F = 2\left\{ B\,\frac{\partial u}{\partial \xi}\Big|_{\xi=0} - \left[2 + \frac{1}{B}\int_0^\infty (1 - u)\,d\xi \right] f_1 \right\}$$

with the boundary conditions

$$u = 0, \ v = 0, \ w = 0 \ \text{ for } \xi = 0;$$
$$u \to 1 \ \text{ as } \ \xi \to \infty.$$

The point $f_1 = 0$, $f_2 = 0$ is a singular point of system (2) and in it the unknown functions must satisfy the one-dimensional equation

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{F}{2B^2}\,\phi\,\frac{\partial u}{\partial \xi} = 0, \tag{3}$$

where

$$F = 2B\,\frac{\partial u}{\partial \xi}\Big|_{\xi=0}; \quad \phi = \int_0^\xi u\,d\xi$$

with the boundary conditions

$$u = 0 \ \text{ for } \xi = 0; \ u \to 1 \ \text{ as } \xi \to \infty.$$

Assuming in system (2) $f_2 = 0$, we get the system

$$
\left.
\begin{aligned}
\frac{Ff_1}{B^2}\, u\,\frac{\partial u}{\partial f_1} + \frac{Ff_1}{B^2}\, v\,\frac{\partial u}{\partial \xi} &= \frac{\partial^2 u}{\partial \xi^2} + \frac{F + 2f_1}{2B^2}\,\phi\,\frac{\partial u}{\partial \xi} + \frac{f_1}{B^2}(1 - u^2); \\
\frac{\partial v}{\partial \xi} + \frac{\partial u}{\partial f_1} &= 0,
\end{aligned}
\right\}
\tag{4}
$$

where

$$F = 2\left\{ B\,\frac{\partial u}{\partial \xi}\Big|_{\xi=0} - \left[2 + \frac{1}{B}\int_0^\infty (1 - u)\,d\xi \right] f_1 \right\}; \quad \phi = \int_0^\xi u\,d\xi$$

with the boundary conditions

$$u = v = 0 \text{ for } \xi = 0; \quad u \to 1 \text{ as } \xi \to \infty.$$

2. Solution of the problem

The problem is solved by the finite differences method.

1. Replacing the derivatives in equation (3) with finite-difference relations, we get the system

$$\frac{u_K^n + 1 - 2u_K^n + u_{K-1}^n}{(\Delta \xi)^2} + \frac{F^{n-1}}{2B^2}\phi_K^{n-1}\frac{u_{K+1}^n - u_{K-1}^n}{2\Delta \xi} = 0; \left.\begin{array}{c} \\ \\ \end{array}\right\} (5)$$

$$F^{n-1} = 2B\frac{-3u_0^{n-1} + 4u_1^{n-1} - u_2^{n-1}}{2\Delta \xi} \quad .$$

with the boundary condition $u_0 = 0$, $u_K = 1$.

Since equation (3) is nonlinear, it is solved by the method of iterations, where the superscript n in system (5) denotes the iteration number. System (5) is reduced to the form

$$a_K^n u_{K-1}^n - 2b_K^n u_K^n + c_K^n u_{K+1}^n = g_K^n, \qquad (6)$$

where

$$a_K^n = 4B^2 - F^{n-1}\phi_K^{n-1}\Delta \xi; \quad b_K^n = 4B^2;$$

$$c_K^n = 4B^2 + F^{n-1}\phi_K^{n-1}\Delta \xi; \quad g_K^n = 0.$$

System (6) is solved by the dispersion method $\underline{/3/}$ with different $\Delta \xi$ and different K such that a further increase in K and a decrease in $\Delta \xi$ with desired accuracy is not reflected in the results. Finally K = 40, $\Delta \xi$ = 0.15 was chosen for the results to agree in the third decimal place.

2. Replacing the derivatives in system (4) with finite-difference relations, we get the system

$$\left(-\frac{F^{n-1}f_1}{B^2}\,u^n\right)_{i+\frac{1}{2},\kappa}\cdot\frac{u^n_{i+1,\kappa}-u^n_{i,\kappa}}{\Delta f_1}+\left(-\frac{F^{n+1}f_1}{B^2}\,v^n\right)_{i+\frac{1}{2},\kappa}\times$$

$$\times\frac{s_2(u_{i+1,\kappa+1}-u_{i+1,\kappa-1})+(1-s_2)(u_{i,\kappa+1}-u_{i,\kappa-1})}{2\Delta\xi}=$$

$$=\frac{s_1(u^n_{i+1,\kappa+1}+u^n_{i+1,\kappa-1}-2u^n_{i+1,\kappa})+(1-s_1)(u^n_{i,\kappa+1}+u^n_{i,\kappa-1}-2u^n_{i,\kappa})}{(\Delta_\eta)^2}+$$

$$+\left(\frac{F^{n-1}+2f_1}{2B^2}\,\phi^{n-1}\right)_{i+\frac{1}{2},\kappa}\times$$

$$\times\frac{s_2(u^n_{i+1,\kappa+1}-u^n_{i+1,\kappa-1})+(1-s_2)(u^n_{i,\kappa+1}-u^n_{i,\kappa-1})}{2\Delta\xi}+$$

$$+\left\{\frac{f_1}{B^2}[1-(u^n)^2]\right\}_{i+\frac{1}{2},\kappa};$$

(7)

$$v^n_{i+1,\kappa+1}=v^n_{i+1,\kappa}-\frac{\Delta\xi}{2}\,\frac{1}{\Delta f_1}\cdot(u^n_{i+1,\kappa+1}+u^n_{i+1,\kappa}-u^n_{i,\kappa+1}-u^n_{i,\kappa});$$

$$F^{n-1}=2\left\{B\,\frac{\partial u}{\partial\xi}\Big|_{\xi=0}-\left[2+\frac{1}{B}\int_0^\infty(1-u^{n-1})\,d\eta\right]f_1\right\}$$

(7)

with the boundary conditions

$$u_0=v_0=0,\quad u_K=1.$$

$$\frac{\partial u^{n-1}}{\partial\xi}\Big|_{\xi=0}=\frac{-3u_0^{n-1}+4u_1^{n-1}-u_2^{n-1}}{2\Delta\xi}.$$

The value of the function at the halfway point equals the arithmetic mean of its values in the neighboring points.

To solve system (7), we take the solution of system (5) as the initial values of the functions u, ϕ, . System (7) reduces to the form

$$a^n_{i,\kappa}u^n_{i+1,\kappa-1}-2b^n_{i,\kappa}u^n_{i+1,\kappa}+c^n_{i,\kappa}u^n_{i+1,\kappa+1}=g^n_{i,\kappa}$$

(8)

where

$$a_{i,\kappa}^n = -s_1 - \frac{s_2 \Delta\xi}{4B^2}\left[(F+2f_1)\Phi\right]_{i+\frac{1}{2},\kappa}^{n-1} + \frac{s_2 \Delta\xi}{2B^2}\left[F^{n-1}f_1 v^n\right]_{i+\frac{1}{2},\kappa};$$

$$2b_{i,\kappa}^n = 2s_1 + \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1}{\Delta f_1}u^n\right)_{i+\frac{1}{2},\kappa};$$

$$c_{i,\kappa}^n = s_1 + \frac{s_2 \Delta\xi}{4B^2}\left[(F+2f_1)\Phi\right]_{i+\frac{1}{2},\kappa}^{n-1} - \frac{s_2 \Delta\xi}{2B^2}\left[F^{n-1}f_1 v^n\right]_{i+\frac{1}{2},\kappa};$$

$$g_{i,\kappa}^n = -\left[(1-s_1) + \frac{(1-s_2)\Delta\xi}{4B^2}\left[(F+2f_1)\Phi\right]_{i+\frac{1}{2},\kappa}^{n-1} - \right.$$
$$\left. - \frac{(1-s_2)\Delta\xi}{2B^2}\left(F^{n-1}f_1 v^n\right)_{i+\frac{1}{2},\kappa}\right]u_{i,\kappa+1}^n +$$
$$+ \left[2(1-s_1) - \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1}{\Delta f_1}u^n\right)_{i+\frac{1}{2},\kappa}\right]u_{i,\kappa}^n -$$
$$- \left[(1-s_1) - \frac{(1-s_2)\Delta\xi}{4B^2}\left[(F+2f_1)\Phi\right]_{i+\frac{1}{2},\kappa}^{n-1} + \right.$$
$$\left. + \frac{(1-s_2)\Delta\xi}{2B^2}\left(F^{n-1}f_1 v^n\right)_{i+\frac{1}{2},\kappa}\right]u_{i,\kappa-1}^n -$$
$$- \frac{(\Delta\xi)^2}{B^2}\left\{f_1[1-(u^n)]^2\right\}_{i+\frac{1}{2},\kappa} \quad (0 < s_1 \leqslant 1, \quad 0 \leqslant s_2 \leqslant 1).$$

system (2) is solved by the dispersion method. The solution is arrived at to the right from the point $f_1 = 0$ to the point at which $F^{n-1} = 0$, and to the left up to the point where $\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} = 0$.

By choosing the values of the parameters s_1 and s_2, we can get different difference schemes. For a given accuracy the most economical scheme, that is, the one that can be used with the largest Δf_1 values, proved to be the scheme which is obtained when $s_1 = s_2 = 0.5$. Automatic selection of the pitch was employed in selecting the Δf_1 values ensuring the given accuracy. Calculation of the coefficients at the halfway points was made iteratively until the results agreed for a given pitch with the given accuracy.

3. Replacing the derivatives in system (2) with finite-difference relations, we get the system

$$\frac{s_1\left(u^n_{i+1,\,\kappa+1,\,j+1} - 2u^n_{i+1,\,\kappa,\,j+1} + u^n_{i+1,\,\kappa-1,\,j+1}\right) + (1-s_1)\left(u^n_{i,\,\kappa+1,\,j+1} - 2u^n_{i,\,\kappa,\,j+1} + u^n_{i,\,\kappa-1,\,j+1}\right)}{(\Delta_\kappa)^2} +$$

$$+ \left(\frac{F^n + 2f_1}{2B^2}\,\varphi^{n-1}\right)_{i+\frac{1}{2},\,\kappa,\,j+1} \times$$

$$\times \frac{s_2\left(u^n_{i+1,\,\kappa+1,\,j+1} - u^n_{i+1,\,\kappa-1,\,j+1}\right) + (1-s_2)\left(u^n_{i,\,\kappa+1,\,j+1} - u^n_{i,\,\kappa-1,\,j+1}\right)}{2\Delta\xi} +$$

$$+ \left[\frac{f_1}{B^2}\left[1-(u^n)^2\right]\right]_{i+\frac{1}{2},\,\kappa,\,j+1} =$$

$$= \left(\frac{c F^{n-1} f_1}{B^2} + f_2\, u^n\right)_{i+\frac{1}{2},\,\kappa,\,j+1} \frac{u^n_{i+1,\,\kappa,\,j+1} - u^n_{i,\,\kappa,\,j+1}}{\Delta f_1} +$$

$$+ \left(\frac{F^{n-1} f_1 + f_2}{B^2}\, v^n\right)_{i+\frac{1}{2},\,\kappa,\,j+1} \times$$

$$\times \frac{s_2\left(u^n_{i+1,\,\kappa+1,\,j+1} - u^n_{i+1,\,\kappa-1,\,j+1}\right) + (1-s_2)\left(u^n_{i,\,\kappa+1,\,j+1} - u^n_{i,\,\kappa-1,\,j+1}\right)}{2\Delta\xi} +$$

$$+ s_4 \left(\frac{(f_1 + 2F^{n-1})f_2}{B^2}\right)_{i+1,\,\kappa,\,j} \left(\frac{u^n_{i+1,\,\kappa,\,j+1} - u^n_{i+1,\,\kappa,\,j}}{\Delta f_2}\, u^n_{i+1,\,\kappa,\,j} + \right.$$

$$\left. + \frac{u^n_{i+1,\,\kappa+1,\,j} - u^n_{i+1,\,\kappa-1,\,j}}{2\Delta\xi}\, w^n_{i+1,\,\kappa,\,j+1}\right) +$$

$$+ s_3 \left(\frac{(f_1 + 2F^{n-1})f_2}{B^2}\right)_{i,\,\kappa,\,j} \left(u^n_{i,\,\kappa,\,j} \frac{u^n_{i,\,\kappa,\,j+1} - u^n_{i,\,\kappa,\,j}}{\Delta f_2} + \right.$$

$$\left. + w^n_{i,\,\kappa,\,j+1} \frac{u^n_{i,\,\kappa+1,\,j} - u^n_{i,\,\kappa-1,\,j}}{2\Delta\xi}\right);$$

$$v^n_{i+1,\,\kappa+1,\,j+1} = v^n_{i+1,\,\kappa,\,j+1} - \frac{\Delta\xi}{2\Delta f_1}\left(u^n_{i+1,\,\kappa,\,j+1} - u^n_{i,\,\kappa,\,j+1} + \right.$$

$$\left. + u^n_{i+1,\,\kappa+1,\,j+1} - u^n_{i,\,\kappa+1,\,j+1}\right);$$

$$w^n_{i+1,\,\kappa+1,\,j+1} = w^n_{i+1,\,\kappa,\,j+1} - \frac{\Delta\xi}{2\Delta f_2}\left(u^n_{i+1,\,\kappa,\,j+1} - u^n_{i+1,\,\kappa,\,j} + \right.$$

$$\left. + u^n_{i+1,\,\kappa+1,\,j+1} - u^n_{i+1,\,\kappa+1,\,j}\right)$$

$$\tag{2}$$

with the boundary conditions

$$u_0 = v_0 = w_0 = 0, \quad u_K = 1.$$

here

$$F^{n-1} = 2\left\{B\,\frac{\partial u^{n-1}}{\partial\xi}\Big|_{\xi=0} - \left[2 - \frac{1}{B}\int_0^\infty (1-u^{n-1})\,d\xi\right]f_1\right\}.$$

- 6 -

The solution of system (7) was adopted for the initial function values when solving system (9).

System (9) is reduced to the form

$$a^n_{i+1,\kappa,j+1} u^n_{i+1,\kappa-1,j+1} - 2b^n_{i+1,\kappa,j} u^n_{i+1,\kappa,j+1} + \tag{10}$$
$$+ c^n_{i+1,\kappa,j+1} u^n_{i+1,\kappa+1,j+1} = g^n_{i+1,\kappa,j+1}$$

where

$$a^n_{i+1,\kappa,j+1} = s_1 - \frac{s_2 \Delta\xi}{4B} [(F+2f_1)\varphi]^{n-1}_{i+\frac{1}{2},\kappa,j+1} +$$

$$+ \frac{s_2 \Delta\xi}{2B^2} [(F^{n-1}f_1 + f_2) v^n]_{i+\frac{1}{2},\kappa,j+1};$$

$$2b^n_{i+1,\kappa,j+1} = 2s_1 + \frac{(\Delta\xi)^2}{B^2} \left(\frac{F^{n-1}f_1 + f_2}{\Delta f_1} u^n \right)_{i+\frac{1}{2},\kappa,j+1} +$$

$$+ \frac{s_4 (\Delta\xi)^2}{B^2} \left(\frac{(f_1 + 2F^{n-1})f_2}{\Delta f_2} u^n \right)_{i+1,\kappa,j};$$

$$c^n_{i+1,\kappa,j+1} = s_1 + \frac{s_2 \Delta\xi}{4B^2} [(F+2f_1)\varphi]^{n-1}_{i+\frac{1}{2},\kappa,j+1} -$$

$$- \frac{s_2 \Delta\xi}{2B^2} [(F^{n-1}f_1 + f_2) v^n]_{i+\frac{1}{2},\kappa,j+1};$$

$$g^n_{i+1,\kappa,j+1} = -(1-s_1)(u^n_{i,\kappa+1,j+1} - 2u^n_{i,\kappa,j+1} + u^n_{i,\kappa-1,j+1}) -$$
$$- \frac{(1-s_2)\Delta\xi}{4B^2} [(F+2f_1)\varphi]^{n-1}_{i+\frac{1}{2},\kappa,j+1}(u^n_{i,\kappa+1,j+1} - u^n_{i,\kappa-1,j+1}) -$$

$$- \frac{(\Delta\xi)^2}{B^2} [f_1(1-u^2)]^n_{i+\frac{1}{2},\kappa,j+1} - \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1 + f_2}{\Delta f_1} u^n\right)_{i+\frac{1}{2},\kappa,j+1} u^n_{i,\kappa,j+1} +$$

$$+ \frac{(1-s_2)\Delta\xi}{2B^2} [(F^{n-1}f_1 + f_2) v^n]_{i+\frac{1}{2},\kappa,j+1}(u^n_{i,\kappa+1,j+1} - u^n_{i,\kappa-1,j+1}) +$$

$$+ s_3 \frac{(\Delta\xi)^2}{B^2} \left[\frac{(f_1 - 2F)f_2}{\Delta f_2} \right]^{n-1}_{i,\kappa,j} \left[u^n_{i,\kappa,j}(u^n_{i,\kappa,j+1} - u^n_{i,\kappa,j}) + \right.$$

$$\left. + w^n_{i,\kappa,j+1} \Delta f_2 \frac{u^n_{i,\kappa+1,j} - u^n_{i,\kappa-1,j}}{2\Delta\xi} \right] +$$

$$+ s_4 \frac{(\Delta\xi)^2}{B^2} \left[\frac{(f_1 - 2F)f_2}{\Delta f_2} \right]^{n-1}_{i+1,\kappa,j} \left[w^n_{i+1,\kappa,j+1} \frac{(u^n_{i+1,\kappa+1,j} - u^n_{i+1,\kappa-1,j})\Delta f_2}{2\Delta\xi} - \right.$$

$$\left. - (u^n_{i+1,\kappa,j})^2 \right].$$

System (10) is solved by the dispersion method. The value of the variable f_1, in which $F_{i,j} f_{1i} + f_{2,j+1} = 0$ was determined in each layer f_{2i}, and the f_1 value was adopted as the starting-point for computation at the $(j + 1)$-th layer. In practical computation, the point where the quantity $F_{i,j} f_{1,i} + f_{2,j+1}$ changes sign was taken as the initial f_1 value. In the first layer, i. e., $j = 1$, $f_1 = 0$ was taken as the initial f_1 value. For the zero layer, the region of integration was found to be bounded by the values $H_2 \leqslant f_1 \leqslant H_1$. In those cases when for $f_2 \neq 0$ the bounds $f = 0$ and $\dfrac{\partial u}{\partial \xi}\Big|_{\xi=0} = 0$ did not exceed the region $|H_1, H_2|$, $s_3 = 0$, $s_4 = 1$ was adopted, which yielded a stable scheme. If $f_2 < 0$ and $f_1 < 0$, then the condition $\dfrac{\partial u}{\partial \xi}\Big|_{\xi=0} = 0$ was satisfied for $f_1 \leqslant H_2$. In this case the region of integration was extended for each layer by one point due to the selection of $s_3 = 1$, $s_4 = 0$ for computation in the layer.

The largest possible Δf_2 value for a given accuracy proved to be $|\Delta f_2| = 0.001$, with which the final calculations were carried out. Some results in the calculations are presented in the table $\left(\zeta = \dfrac{\partial u}{\partial \xi}\Big|_{\xi=0}\right)$.

All calculations were performed on the BESM-3 computer of the LOTsEI [expansion unknown] of the USSR Academy of Sciences.

Values of the quantities f and ... in the ... and ... approximation

$f_3 = 0$			$f_3 = 0.01$			$f_3 = 0.01$			$f_3 = 0.02$		
f_1	F		f_1	F		f_1	F		f_1	F	
0,085	−0,0124	0,3535	0,0825	−0,0010	0,3494	0,08125	−0,0005	0,3430	0,07525	−0,0055	0,3270
0,08	0,0235	0,3511	0,08	−0,0091	0,3460	0,08	0,0056	0,3415	0,075	0,0012	0,3231
0,07	0,0706	0,3350	0,075	0,0332	0,3384	0,075	0,0325	0,3314	0,07	0,0274	0,3151
0,06	0,1196	0,3190	0,07	0,0578	0,3302	0,07	0,0586	0,3272	0,06	0,0694	0,3015
0,05	0,1702	0,3031	0,06	0,1080	0,3146	0,06	0,1110	0,3127	0,05	0,1311	0,2855
0,04	0,2220	0,2870	0,05	0,1597	0,2989	0,05	0,1632	0,2978	0,04	0,1886	0,2715
0,03	0,2752	0,2709	0,04	0,2131	0,2833	0,04	0,2152	0,2822	0,03	0,2439	0,2560
0,02	0,3295	0,2545	0,03	0,2691	0,2680	0,03	0,2674	0,266?	0,02	0,3020	0,2403
0,01	0,3850	0,2379	0,02	0,3298	0,2541	0,02	0,3211	0,2493	0,01	0,3570	0,2241
0	0,4416	0,2208	0,01	0,4054	0,2448	0,01	0,3761	0,2333	0	0,4119	0,2074
−0,01	0,4998	0,2034	0	0,4727	0,2363	0	0,4332	0,2166	−0,01	0,4738	0,1902
−0,02	0,5591	0,1855	−0,01	0,5160	0,2128	−0,01	0,4912	0,1995	−0,02	0,5339	0,1724
−0,03	0,6197	0,1668	−0,02	0,5732	0,1949	−0,02	0,5503	0,1817	−0,03	0,5951	0,1537
−0,04	0,6817	0,1471	−0,03	0,6311	0,1759	−0,03	0,6109	0,1633	−0,04	0,6578	0,1339
−0,05	0,7454	0,1263	−0,04	0,6910	0,1564	−0,04	0,6732	0,1442	−0,05	0,7221	0,1125
−0,06	0,8110	0,1036	−0,05	0,7528	0,1359	−0,05	0,7374	0,1243	−0,06	0,7855	0,0890
−0,07	0,8792	0,0780	−0,06	0,8166	0,1139	−0,06	0,8037	0,1026	−0,07	0,8578	0,0622
−0,08375	0,9789	0,0325	−0,07	0,8827	0,0899	−0,07	0,8711	0,0774	−0,08	0,9324	0,0293
−0,09	1,0266	−0,0051	−0,08	0,9518	0,0621	−0,08	0,9120	0,0171	−0,085	0,9713	0,0011
			−0,085	0,9879	0,00157	−0,085	0,9793	0,0280	−0,08925	0,9812	−0,0048
			−0,09	1,025	0,0259	−0,09	1,018	0,0018			

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[continuation 3007]

I_1	$I_2 = -0.01$		I_1	$I_3 = -0.03$		I_1	$I_2 = -0.07$		I_1	$I_3 = -0.1$	
	f	$\cdot$		f	z		f	$\cdot$		f	$\cdot$
0.06	0.0936	0.3799	0.01	0.2791	0.3119	0.03	0.2985	0.2611	0.03	0.3012	0.2523
0.05	0.0616	0.3491	0.03	0.2913	0.2793	0.02	0.3463	0.2713	0.02	0.3765	0.2755
0.06	0.1285	0.3225	0.02	0.3574	0.2672	0.01	0.4286	0.2555	0.01	0.4353	0.2502
0.05	0.1778	0.3061	0.01	0.4151	0.2522	0	0.4939	0.2469	0	0.5026	0.2515
0.01	0.2286	0.2898	0	0.4771	0.2386	-0.01	0.5169	0.2291	-0.01	0.5665	0.2354
0.03	0.2820	0.2739	-0.01	0.5290	0.2189	-0.02	0.6058	0.2118	-0.02	0.6196	0.2194
0.02	0.3361	0.2576	-0.02	0.5833	0.2002	-0.03	0.6636	0.1932	-0.03	0.6789	0.2023
0.01	0.3909	0.2407	-0.03	0.6438	0.1813	-0.01	0.7223	0.1738	-0.01	0.7357	0.1649
0	0.4516	0.2258	-0.01	0.7042	0.1620	-0.03	0.7832	0.1536	-0.05	0.7993	0.1444
-0.01	0.5058	0.2067	-0.03	0.7666	0.1418	-0.06	0.8461	0.1323	-0.06	0.8611	0.1233
-0.02	0.5670	0.1888	-0.06	0.8308	0.1204	-0.07	0.9113	0.1099	-0.07	0.9219	0.0982
-0.03	0.6257	0.1705	-0.07	0.8974	0.0972	-0.08	0.9788	0.0631	-0.08	0.9905	0.0703
-0.01	0.6875	0.1511	-0.08	0.9660	0.0714	-0.09	1.0496	0.0355	-0.09	1.0556	0.0518
-0.05	0.7510	0.1306	-0.09	1.0101	0.0091	-0.095	1.0668	0.0378	-0.095	1.0955	0.0355
-0.06	0.8162	0.1043	-0.0925	1.0800	0.0291	-0.1	1.1254	0.0154	-0.1	1.1331	0.0143
-0.07	0.8839	0.0838	-0.095	1.0743	0.0172	-0.1025	1.1442	-0.001	-0.105	1.1722	-0.0024
-0.08	0.9519	0.0551	-0.0975	1.0973	0.0013	—	—	—	-0.1075	1.1908	—
-0.09	1.0301	0.0137	—	—	—	—	—	—	—	—	—
-0.0925	1.0192	-0.0018	—	—	—	—	—	—	—	—	—

t_1	$t_2=-0.11$		t_1	$t_2=-0.15$		t_1	$t_2=-0.18$		t_1	$t_2=-0.22$	
	r	z		r	z		r	z		r	z
0,04	0,3039	0,2633	0,03	0,3098	0,2815	0,01	0,3090	0,2551	0,01	0,3212	0,6??
0,02	0,3745	0,2751	0,02	0,3786	0,2709	0,02	0,3815	0,2783	0,02	0,3544	?,??
0,01	0,4109	0,2615	0,01	0,4???	0,2620	0,01	0,4590	0,2555	0,01	0,4533	0,5??
0	0,5009	0,2550	0	0,5157	0,2378	0	0,5203	0,2702	0	0,5342	0,2?21
−0,01	0,5685	0,2397	−0,01	0,5732	0,2432	−0,01	0,5899	0,2461	−0,01	0,5557	0,2453
−0,02	0,6290	0,2215	−0,02	0,6366	0,2287	−0,02	0,6631	0,2???	−0,02	0,6455	0,2333
−0,03	0,6897	0,2086	−0,03	0,6?49	0,2135	−0,03	0,706?	0,???	−0,03	0,7116	0,2213
−0,04	0,7507	0,1907	−0,04	0,7600	0,1974	−0,04	0,7678	0,2022	−0,04	0,7746	0,6??
−0,05	0,8122	0,1737	−0,05	0,8272	0,1801	−0,05	0,8306	0,1859	−0,05	0,8378	0,19??
−0,06	0,8745	0,1543	−0,06	0,8651	0,1622	−0,06	0,8609	0,1783	−0,06	0,9016	0,17??
−0,07	0,9369	0,1334	−0,07	0,9170	0,1428	−0,07	0,9382	0,1500	−0,07	0,9661	0,1551
−0,08	1,003	0,1110	−0,08	1,014	0,1215	−0,08	1,023	0,1299	−0,08	1,031	0,1369
−0,09	1,0702	0,0831	−0,09	1,0605	0,0976	−0,09	1,0997	0,1073	−0,09	1,0977	0,1156
−0,095	1,1052	0,0707	−0,095	1,1115	0,0811	−0,095	1,1237	0,0831	−0,095	1,1310	0,1042
−0,1	1,1013	0,0517	−0,1	1,1500	0,0702	−0,1	1,1581	0,0824	−0,1	1,1661	0,0221
−0,105	1,1789	0,0351	−0,105	1,1863	0,0513	−0,105	1,1940	0,0692	−0,105	1,2013	0,0792
−0,11	1,2181	0,0156	−0,11	1,2210	0,0360	−0,11	1,2305	0,0525	−0,11	1,2373	0,0350
−0,115	1,2369	−0,0025	−0,115	1,2532	0,0131	−0,115	1,2555	0,0312	−0,115	1,2743	0,0491
−0,1125	—	—	−0,1175	1,2823	−0,0025	−0,12	1,303	0,0110	−0,12	1,313	0,0305
	—	—	−0,1225	—	—	−0,1225	1,327	−0,0032	−0,1225	1,332	0,0194
	—	—		—	—		—	—	−0,125	1,352	0,0053
	—	—		—	—		—	—	−0,12625	1,362	−0,0016